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IMPROVED COUPLED-MODE THEORY OF PARALLEL OPTICAL WAVEGUIDES

Hani A. Harb[1]

[1]Department of Computer Science and Engineering
Faculty of CS &IT, Al-Baha University, Saudi Arabia.

ABSTRACT

In this paper the transverse coupling between coupled parallel two dielectric slab waveguides (slab-slab) is analyzed for the first time –to the author's knowledge by Improved Coupled- Mode Theory (ICMT). The coupling is established via the evanescent part of the modal field propagating in one waveguide which penetrates the other waveguide. The overlap integrals, coupling coefficients, asynchronism factor, output powers and coupling lengths are evaluated numerically by ICMT, where the results obtained reveal an accuracy improvement over that obtained by the conventional coupled mode theory (CMT).

Keywords: Improved -coupled mode theory, Coupled -mode theory, Evanescent wave coupling, Optical coupling .

1- INTRODUCTION

Evanescent wave coupling phenomena is widely used to verify efficient light coupling between two closely – spaced waveguides fabricated side by side or by utilizing diffraction gratings [1-5].

Electromagnetic waves propagating in a dielectric waveguide composed of a set of independent modes, the modes of two dielectric waveguides can exchange energy if they propagate at equal or nearly equal propagation constants. The presence of a waveguide near another can be considered as a perturbation of one guide by the other and vice versa. An approximate solutions based on Coupled-Mode Theory [6-11] and Improved coupled –mode Theory are used to analyze mode coupling phenomena rather than the exact solution by solving Maxwell's equations governing electromagnetic fields and its related boundary conditions. The coupled-mode theory (CMT) is the most representative analytical approach based on the overlap integrals of evanescent fields [12], [13]. The coupled-mode formulations are correct within the first-order of perturbation, the Improved Coupled-Mode Theory (ICMT) offers second-order corrections to (CMT). Both approaches are derived from Lorentz's reciprocity theorem[14]. Coupling in CMT & ICMT can be represented by two coupled differential equations of first order describing coupling behavior. Coupling phenomena is analyzed in coupled parallel two dielectric slab waveguides by CMT & ICMT.

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2 - COUPLED PARALLEL DIELECTRIC WAVEGUIDES

The transverse components of each field expanded in terms of

The author is with Department of Electronics and Communications, Mansoura University, Egypt. (e-mail: ***haniharb0@gmail.com***,).

a finite set of guided normal modes with discrete propagation constants and an infinite integration set of radiation modes with a continuous spectrum. Guided normal modes of waveguide determined from the solution of scalar wave equation. The electromagnetic field components for each waveguide in isolation in terms of modal amplitudes and modal profiles respectively expressed as follows

$$E = \sum_{k=1}^{N}\left[A_k(z)e^{-j\beta_k z} + A_{-k}(z)e^{j\beta_k z}\right]e_{kt} + \int_{-\infty}^{\infty} A(\beta,z)e^{j\beta z}e_{rt}\,d\beta \quad (1)$$

$$H = \sum_{k=1}^{N}\left[A_k(z)e^{-j\beta_k z} + A_{-k}(z)e^{j\beta_k z}\right]h_{kt} + \int_{-\infty}^{\infty} A(\beta,z)e^{j\beta z}h_{rt}\,d\beta \quad (2)$$

Radiation modes (second term) could be ignored as coupling of power between guided modes is the main interest. We assume that each waveguide in isolation supports only one mode. The perturbation in the transverse direction is due to the presence of adjacent waveguides [9], [12]. The electromagnetic fields for composite waveguides shown in Fig.1 may be approximated by weighted sum of the unperturbed fields as follows

$$E(x,y,z) = a(z)E^{(a)}(x,y) + b(z)E^{(b)}(x,y) \quad (3)$$

$$H(x,y,z) = a(z)H^{(a)}(x,y) + b(z)H^{(b)}(x,y) \quad (4)$$

Where, E(x, y) and H(x, y) are unperturbed fields modal profile functions in the transverse direction in waveguides a and b (i.e. fields of individual waveguide in isolation), a(z) and b(z) are the modal amplitudes in z-direction[14].

3- COUPLED PARALLEL OPTICAL PLANAR (SLAB) WAVEGUIDES

Two dielectric slab waveguides placed next to each other. Each waveguide composed of a slab of high refractive index surrounded by two slabs of lower index as shown in Fig.1. Numerical example is used to analyze coupling phenomena in coupled slab waveguides by ICMT and CMT at weakly guiding , where two coupled slab waveguides shown in Fig.1 have refractive indices n_a=2.2 & n_b=2.2 surrounded by medium extending infinitely with refractive index n_s=2.19, the wave propagating wavelength λ is 1.6 μm, the waveguide dimensions are d_a=d_b=d=2 μm and edge to edge separation t=1.9 μm**.** Refractive index difference Δn introduced between waveguides a and b by applying external voltage across the two waveguides such that,

$$\Delta n = n_a - n_b,\ n_a = 2.2 + \Delta n/2,\ n_b = 2.2 - \Delta n/2\ .$$

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The composite waveguide is uniform in the y-direction and the permeability of free space is assumed over the whole area]. We consider TE_0-TE_0 mode coupling between waveguides, The wave functions $E_y^a(x,y)$ and $E_y^b(x,y)$ are the y-components of the electric field of the TE_0 mode propagating in z direction in waveguides a and b respectively which is the modal profile in the two waveguides and could be given as follows[15]

$$E_y^a(x,y)=\begin{cases} Ae^{V\sqrt{b}(x/d)} & ,\text{for } x\le 0 \\ A\left[\cos(V\sqrt{1-b}\,\frac{x}{d})+\sqrt{\frac{b}{1-b}}\sin(V\sqrt{1-b}\,\frac{x}{d})\right] & ,\text{for } 0\le x\le d \\ A\left[\cos(V\sqrt{1-b})+\sqrt{\frac{b}{1-b}}\sin(V\sqrt{1-b})\right]e^{-V\sqrt{s+b}(x-d)/d} & ,\text{for } x\ge d \end{cases} \quad (5)$$

$$E_y^b(x,y)=\begin{cases} Ae^{V\sqrt{b}(x-t_1/d)} & ,\text{for } x\le t_1 \\ A\left[\cos(V\sqrt{1-b}\,\frac{(x-t_1)}{d})+\sqrt{\frac{b}{1-b}}\sin(V\sqrt{1-b}\,\frac{(x-t_1)}{d})\right] & ,\text{for } t_1\le x\le d+t_1 \\ A\left[\cos(V\sqrt{1-b})+\sqrt{\frac{b}{1-b}}\sin(V\sqrt{1-b})\right]e^{-V\sqrt{s+b}(x-t_1-d)/d} & ,\text{for } x\ge t_1+d \end{cases} \quad (6)$$

Where, v is the generalized frequency, b is the normalized guide index, A is determined such that the power flow normalized to unity such that $\frac{\beta}{2\omega\mu}\int_{-\infty}^{\infty}\left[E_y\right]^2=1$, where β is the propagation constant of the given propagating mode, ω is propagating wave angular frequency.

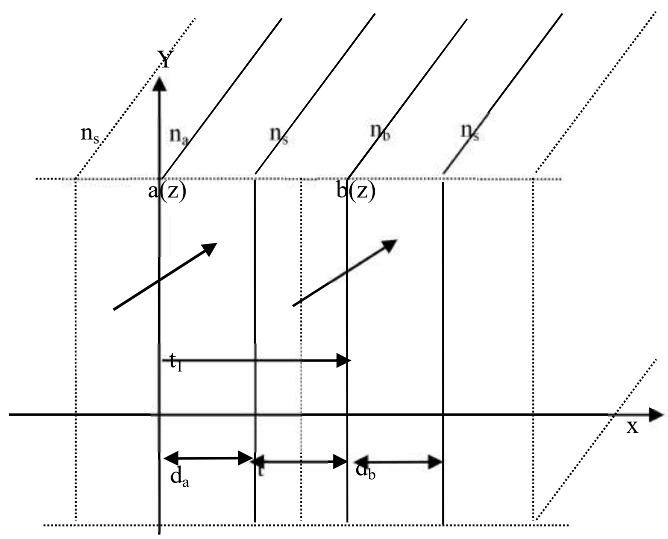

Fig.1 Geometry for coupled dielectric slab waveguides.

4- COUPLING PARAMETERS

Overlap integrals, Coupling coefficients, Asynchronism factor, Output powers and Coupling length in waveguides (a) and (b) are evaluated by both the CMT and ICMT at weakly guiding where, the difference between the refractive indices are so small and at strong guiding where, the difference is greater. The effect of

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distance between coupled parallel waveguide was taken into consideration. The fundamental modes are initiated at waveguide(b). The computations of the integrals are done numerically.

4-1 Overlap Integrals

Overlap integrals identifies evanescent wave coupling between waveguides and count for the coupling coefficients asymmetry or symmetry, also the coupling degree between two waveguide modes [9].
The overlap integrals (C_{ab} and C_{ba}) between waveguides (a) and (b) are given as follows [13],

$$C_{ab} = \frac{N_a}{2\eta_o} \iint (E_y^b * E_y^a) \hat{k} \cdot \hat{k} \, dxdy \qquad (7)$$

$$C_{ba} = \frac{N_b}{2\eta_o} \iint (E_y^a * E_y^b) \hat{k} \cdot \hat{k} \, dxdy \qquad (8)$$

Where, N_a, N_b effective refractive indices in waveguides (a) and (b) respectively and η_o is the intrinsic impedance of free space.
Where,

$H_x^a = -\frac{N_a}{\eta_o} E_y^a$, $H_y^a \approx 0$ and $H_x^b = -\frac{N_b}{\eta_o} E_y^b$, $H_y^b \approx 0$. The average overlap integral C is given by

$$C = \frac{1}{2}(C_{ab} + C_{ba}) = \frac{(N_a + N_b)}{4\eta_o} \iint (E_y^a * E_y^b) \hat{k} \cdot \hat{k} \, dxdy \qquad (9)$$

Average overlap integrals are symmetric with the symmetric induced refractive index difference. Increasing outside refractive index n_s=2.195 (weakly guiding) would increase average overlap integrals. Decreasing outside refractive index n_s=2.19 (strong guiding) would decrease average overlap integrals as shown in Fig.2. Variation in average overlap integral in strong guiding is (0.0096) which is greatly less than that of weakly guiding which is (0.0382).

4-2 Coupling coefficients

Coupling coefficient K_{ab} between waveguides (a) and (b) and coupling coefficient K_{ba} between waveguides (b) and (a) are evaluated for the coupled parallel slab waveguides .
Cross coupling coefficients K_{ab} and K_{ba} between waveguides(a) and (b) given by CMT are given as follows[6], [14],

$$K_{ab} = \frac{\omega}{4} \iint \Delta\varepsilon^b \left[E_y^a . E_y^b\right] dxdy \qquad (10)$$

$$K_{ba} = \frac{\omega}{4} \iint \Delta\varepsilon^a \left[E_y^b . E_y^a\right] dxdy \qquad (11)$$

Where,

$E_x^a = 0, E_z^a = 0$ and $E_x^b = 0, E_z^b = 0$,

$\Delta\varepsilon^a = \varepsilon - \varepsilon_a = \varepsilon_b$,

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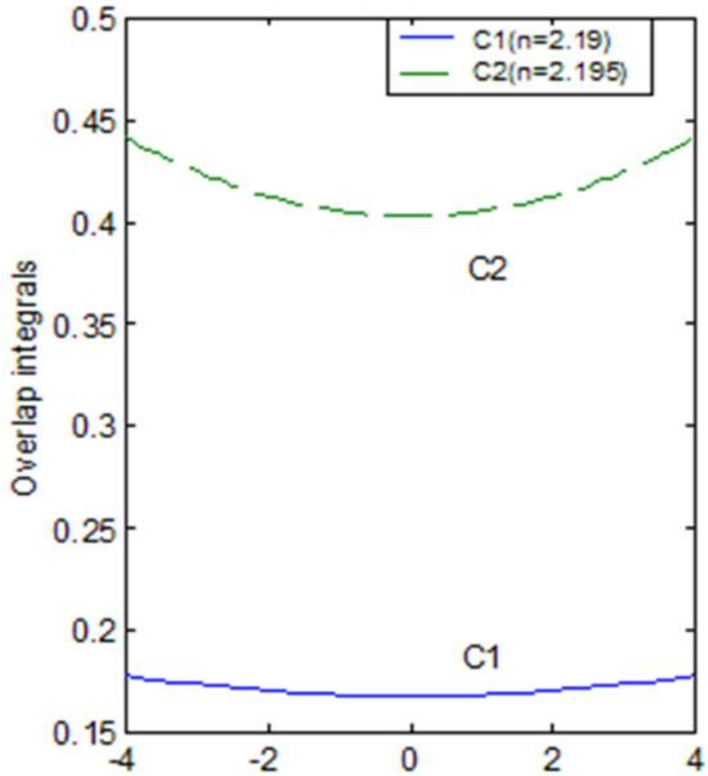
$\Delta \varepsilon^{b} = \varepsilon - \varepsilon_{b} = \varepsilon_{a}$



Refractive index difference between
cores of waveguides a and b in 1.e-3

Fig.2 Average overlap integrals versus refractive index difference between waveguides cores in 1.e-3 at outside refractive indices 2.19(C1), 2.195(C2).

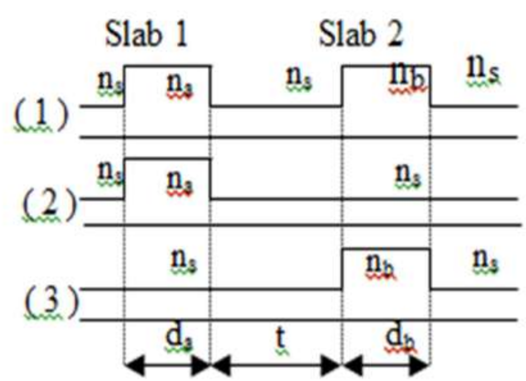


Fig.3 Coupled slab waveguides, the dielectric profile of (1) The waveguide system. (2) Waveguide (a). (3) Waveguide (b).

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The dielectric profile for the waveguide system and each waveguide in isolation are shown in Fig.3.

The coupling coefficients k_{ab} and k_{ba} given by ICMT are given by [13]

$$k_{ab} = \frac{K_{ab} - CK_{bb}}{1 - C^2} \quad (12)$$

$$k_{ba} = \frac{K_{ba} - CK_{aa}}{1 - C^2} \quad (13)$$

Where, self coupling coefficients K_{aa}, K_{bb} are given as follows

$$K_{aa} = \frac{\omega}{4} \iint \Delta\varepsilon^{a} \left[E_y^a . E_y^a\right] dxdy \quad (14)$$

$$K_{bb} = \frac{\omega}{4} \iint \Delta\varepsilon^{b} \left[E_y^b . E_y^b\right] dxdy \quad (15)$$

Where, $E_x^a = 0, E_z^a = 0$ and $E_x^b = 0, E_z^b = 0$

The coupling coefficients K_{ab} & K_{ba} increased noticeably at weakly guiding (n_s=2.195) more than those in trong guiding (n_s=2.19) as shown in Fig.4. The coupling coefficients between waveguides given by ICMT & CMT are almost the same at strong guiding (n_s=2.19), while the coupling coefficients given by ICMT are greater than coupling coefficients given by CMT at weakly guiding (n_s=2.195), where the coupling coefficients are well separated.

The coupling coefficients between coupled waveguides decreased with increasing distance between waveguides as shown in Fig.5. In coupled slab waveguides, at lower distance the coupling coefficients given by ICMT is obviously separated and greater than that given by CMT, while at greater distances the coupling coefficients given by ICMT & CMT are nearly the same. The difference between coupling coefficients at weakly guiding is greater than that given at strong guiding as shown in Fig.5.

4-3 ASYNCHRONISM FACTOR δ

Asynchronism factor expresses the point of maximum power transfer between waveguides, also the effect of propagating modes propagation constants deviations from each other [9]. Asynchronism factor given by CMT and ICMT could be given by $\delta = \frac{\beta_b - \beta_a}{2\sqrt{K_{ab}K_{ba}}}$, $\delta = \frac{\gamma_b - \gamma_a}{2\sqrt{k_{ab}k_{ba}}}$ respectively. Where,

$$, \gamma_a = k_o N_a + \frac{K_{aa} - CK_{ba}}{1 - C^2}$$

$$\gamma_b = k_o N_b + \frac{K_{bb} - CK_{ab}}{1 - C^2}$$

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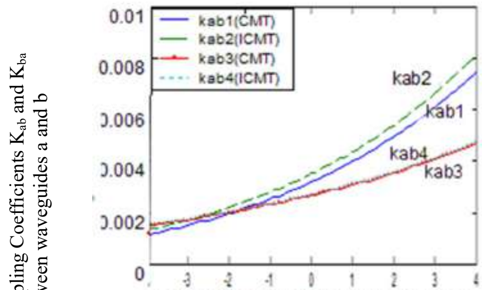
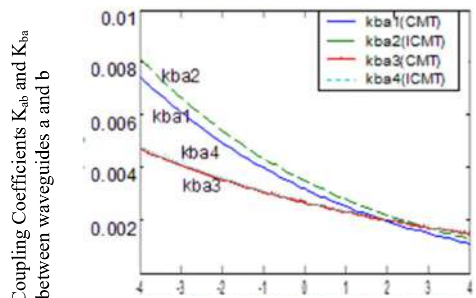


(a) Refractive index difference between cores of waveguides a and b in 1.e^{-3}



(b) Refractive index difference between cores of waveguides a and b in 1.e^{-3}

Fig.4 (a) Coupling coefficient K_{ab} in μm^{-1}, (b) Coupling coefficient K_{ba} in μm^{-1} versus refractive index difference at n_s=2.195, n_s=2.19 by CMT & ICMT.

Asynchronism factors between two parallel slab waveguides are evaluated by ICMT & CMT for weakly and strong guiding. It is found that this factors are almost the same at strong guiding (n_s=2.19) as shown in Fig.6(AS3,AS4), while that given by ICMT is lower than Asynchronism factor given by CMT in weakly guiding (n_s=2.195)as shown in Fig.6(AS1,AS2), where, Asynchronism factor are well separated.

Increasing the surrounding refractive index (weakly guiding) would decrease the Asynchronism factor slope (i.e. the propagation constant difference get closer to zero value), then power exchange between waveguides is nearly complete as shown in Fig.6

4-4 Output Powers

Output powers P_a and P_b in waveguides (a) and (b) respectively when one of them terminates the other at z= L are given by

$$P_a = 1 - \left(\frac{1 - C_{ab}C_{ba}}{1+\delta^2}\right) e^{2\sinh^{-1} C\delta} \sin^2\left[\sqrt{K_{ab}K_{ba}}\, L\left(1+\delta^2\right)^{1/2}\right] \qquad (16)$$

$$P_b = C_{ab}C_{ba} + \left(\frac{1 - C_{ab}C_{ba}}{1+\delta^2}\right) \sin^2\left[\sqrt{K_{ab}K_{ba}}\, L\left(1+\delta^2\right)^{1/2}\right] \qquad (17)$$

Minimum power transfer in waveguide (a) by ICMT occurs at the right of minimum given by CMT(at zero refractive index difference)

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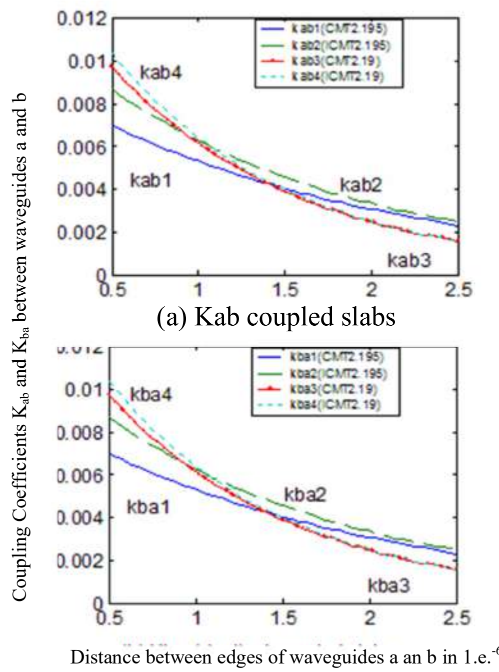
due to cross talk. In strong guiding (n_s=2.19)

output power P_a in waveguide (a) are almost the same for CMT&ICMT but with small deviation to the right as shown in Fig.7(a), while weakly guiding(n_s=2.195) will make the output power spread more than strong guiding and more deviation to the right as shown in Fig.7(b). Maximum power transfer in waveguide (b) occurs at zero refractive index difference.



(b) Kba coupled slabs

Fig.5 coupling coefficients K_{ab} and K_{ba} in □m versus distance between core's edges of coupled waveguidse in □m by CMT and ICMT at (a) n_1 = 2.195, (b) n_1 = 2.19, for slab waveguides.

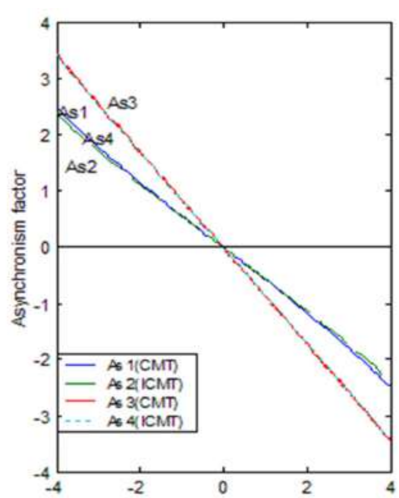


Fig.6 Asynchronism factor versus refractive index difference at n_s=2.195, n_s=2.19 by CMT & ICMT.

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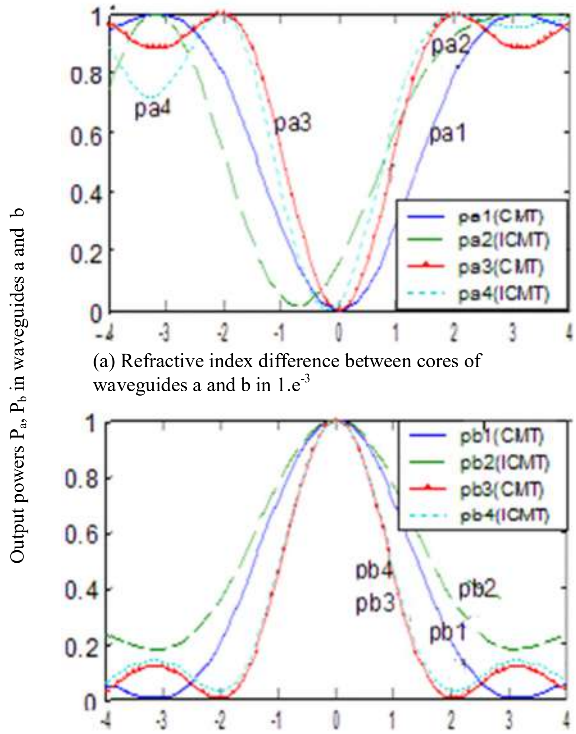

(a) Refractive index difference between cores of waveguides a and b in $1.e^{-3}$

(b) Refractive index difference between cores of waveguides a and b in $1.e^{-3}$

Fig.7 (a), (b) Output powers P_a, P_b in waveguides a and b respectively versus refractive index difference at n_s=2.195, n_s=2.19 by CMT & ICMT.

4-5 Coupling Length

The coupling length L at maximum power coupling (synchronous coupling) occurs at Δ=0 which is the phase matching condition [6], [9]. We have $\psi = \sqrt{\Delta^2 + K_{ab}K_{ba}}$ and $\Delta = \frac{\gamma_b - \gamma_a}{2}$.

The coupling length in case of synchronous coupling is given by

$L = \pi / 2\psi$, $\psi = \sqrt{K_{ab}K_{ba}}$, $L = \pi / 2\sqrt{K_{ab}K_{ba}}$ (18)

The coupling length at incomplete power coupling (asynchronous coupling) $\Delta \neq 0$ is given by

$L = \pi / 2\psi$, $L = \pi / \left(2\sqrt{\Delta^2 + K_{ab}K_{ba}} \right)$ (19)

The coupling lengths given by CMT and ICMT are,

$\pi / 2\sqrt{K_{ab}K_{ba}}$ and $\pi / 2\sqrt{k_{ab}k_{ba}}$ respectively.

In strong guiding (n_s=2.19), the coupling length is slightly reduced from 0.5902 mm to0.5811 mm given by CMT and ICMT respectively. In weakly guiding (n_s=2.195), the coupling length is reduced from 0.4925 mm to

0.448 mm given by CMT and ICMT respectively. The coupling length greatly reduced at weakly guiding, ICMT generally reduce the coupling length below the coupling length given by CMT.

5. CONCLUSION

The results of the ICMT reveals an accuracy improvement than those obtained by the conventional coupled mode theory (CMT), which is considered to be second order correction to CMT. Increasing outside medium refractive index (weakly guiding) would increase overlap integrals and coupling coefficients and vice versa. The variation in average overlap integral at (weakly guiding) is much greater than at (strong guiding).

Strong coupling couldn't affect the slope of Asynchronism factor greatly as the effect of outside refractive index. Weakly guiding will make the output power spread more than strong guiding and more deviation to the right. Crosstalk is increased at weakly guiding more than strong guiding. The coupling length greatly reduced at weakly guiding.

Coupling coefficients in slab waveguides given by ICMT are greater than those given by CMT in weakly guiding. At weakly guiding the asynchronism factor slope is decreased (i.e. the propagation constant difference get closer to zero value), asynchronism factor given by ICMT & CMT are almost the same at strong guiding, while at weakly guiding the asynchronism factor given by ICMT is lower than that given by CMT. The power exchange between waveguides is completed. Minimum power transfer by ICMT occurs at the right of minimum given by CMT due to cross talk. At strong guiding the right refractive index difference shift given by ICMT over CMT is uncensored while this shift is obvious at weakly coupling. ICMT generally reduce the coupling length below the coupling length given by CMT. At lower distance between coupled waveguides, the coupling coefficients given by ICMT is obviously larger than that given by CMT, while at greater distances coupling coefficients given by ICMT & CMT are nearly the same.

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